Karunya University
(Karunya Institute of Technology and Sciences)
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End Semester Examinations - 2015-16 Even Semester - May 2016

14EI3008 Optimal Control Theory

Set B

Time : 3 hrs
Total Marks: 100

1. a. Illustrate the selection of a performance measure to design an automatic control system for landing a high speed jet airplane on the deck of an aircraft carrier. (15)
b. Explain the characteristics of Dynamic Programming. (5)

OR

2. a. A first order system is described by the differential equation (15)
$\dot{x}(t)=x(t)+u(t)$;
Find the control law that minimizes the performance measure
$$J=(1/4)x^2(T)+\int_0^T(1/4)u^2(t)dt$$
The final time T is specified and the admissible state and control values are not constrained by any boundaries.
b. What are the advantages of state variable representation? (5)

3. a. Draw the flowchart which depicts the computational procedure for solving control problems.(10)
b.Explain the various performance measures for optimal control problems.(10)

OR

4. a. Derive the Euler Lagrange Equation (10)
b. Determine an extremal for the functional
$J(x) = \int_0^2[\dot{x}^2(t)+2x(t)\dot{x}(t)+4x^2(t)]dt$; x (0)=1 and x(2) is free. (10)

5. a. Derive the Hamilton-Jacobi-Bellman Equation. (10)

b. Find an extremal for the functional $J(x) = \int_1^{tf}\left[2x(t)+\frac{1}{2}\dot{x}^2(t)\right]$; the boundary conditions are $x(1)=4$, $x(t_f)=4$, and $t_f > 1$ is free. (10)

OR

6. a. Derive ARE (Algebraic Riccati Equation) for linear tracking problem. (10)
b. b. Consider the system described by its state equation (10)
$$\dot{x}1(t)=x2(t)$$
$$\dot{x}2(t)=-x2(t)+u(t)$$
with initial conditions $x(t0)=x_0$. The performance measure to be minimized is
$J(u)=\int_{t0}^{tf}\frac{1}{2}[x_1{}^2+u^2(t)]\,dt$;
tf is specified, and the final state $x(t_f)$ is free.
i) Find necessary conditions for an unconstrained control to minimize J.
ii) Find necessary conditions for optimal control if $-1\le u(t)\le +1$

7. a.The system $\dot{x}1(t)=x2(t)$
$\dot{x}2(t)=2x1(t)-x2(t)+u(t)$
is to be controlled to minimize the performance measure
$J(u)=[x1(T)-1]^2+\int_0^T\{[x1(t)-1]^2+0.0025u^2(t)\}dt$.
The final time T is specified, x(T) is free, and the admissible states and controls are not bounded.
Determine the optimal control law. (10)
b. Determine the form of the optimal control for a class of minimum-time problems. (10)

OR

8.

a. Find a necessary condition that must be satisfied by an extremal of the functional

$$F(x)=\int_{t_0}^{t_1} g(x,\dot{x},t)\,dt$$ where t_0, $x(t_0)=x_0$ are specified and t_f and $x(t_f)$ are free. (10)

b. Find an extremal curve for the functional $J(x)=\int_{t0}^{tf}[1+\dot{x}^2(t)]^{1/2}\,dt$; the boundary conditions $t_o=0$, $x(0)=0$ are specified, t_f and $x(t_f)$ are free, which begins at the origin and terminates on the curve $\theta(t)=\frac{1}{2}[t-5]^2-\frac{1}{2}$ (10)

9.

a) Discuss the method of Steepest Descent(or gradients)used for minimization of Functions.(10)

b)Compare the features of 3 iterative methods for solving non-linear 2-point boundary value problems. (10)

Wishing you All the Best
